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Improved Physics Model for the Gasdynamic Mirror Fusion Propulsion System

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The propulsion capability of the gasdynamic mirror (GDM) fusion propulsion device was examined in several previous publications without taking into account the electrostatic potential inherent to plasma confinement in this system. This potential arises as a result of the initial rapid escape of the electrons through the mirrors because of the smallness of their mass. The remaining excess positive charge gives rise to a positive electric potential that slows down the electrons while speeding up the ions until equalization in their axial diffusion is achieved. In a thruster, the energy of the ions emerging from the magnetic nozzle will therefore be enhanced relative to their energy as they leave the mirror by an amount equal to that of the potential. In typical GDM parameters, this effect can translate into significant increases in the specific impulse and thrust produced by the system.

Nomenclature

A_c = area of plasma core
A_0 = mirror area
D = axial diffusion coefficient
E = electric field
E_e = electron energy
E_L = escape energy
e = electron charge
erf = error function
k = gradient scale length
L = length of plasma
$\ell n\ \lambda$ = coulomb logarithm
m = particle mass
N = particle density
n = monoenergetic particle density
R = plasma mirror ratio
T = temperature
V = plasma volume
v = monoenergetic particle velocity
v_{th} = thermal velocity
x = parameter, Eq. (26)
z = charge number
Γ = velocity-averaged particle flux
γ = monenergetic flux
δ = parameter, Eq. (25)
μ = mobility
τ = confinement time
υ = collision frequency
ϕ = electrostatic potential

Introduction

THE gasdynamic mirror (GDM) fusion propulsion system (Fig. 1) is a magnetic mirror confinement system in which a hot dense plasma is confined long enough to allow fusion reactions to take place while allowing a fraction of the charged particles to escape to produce the desired thrust.[1] Because the collision mean free path in this device is much shorter than the length, the plasma behaves much like a fluid with confinement properties dictated by gasdynamic laws. Under these circumstances, the escape of plasma from the system is analogous to the flow of a gas into vacuum from a vessel with a hole. As a thruster, GDM will operate as an asymmetric mirror where a fraction of the charged particle power dictated by an appropriate mirror ratio will emerge through one mirror to generate the thrust, while the remainder is allowed to escape through the opposite mirror to a direct converter to convert its energy to electric power.[2] Additional power is generated through a thermal converter that processes the radiation and neutron power generated by the fusion reactions, and when the two are combined they are supplied to the injector, which, in turn, supplies the needed power to heat the plasma in the reactor chamber to thermonuclear temperatures. With the aid of a power flow diagram, it has been shown that a magnification factor Q (defined as the ratio of fusion power to injected power), with a value slightly larger than unity, is adequate for the system to be self-sustaining.[2] This presupposes certain efficiencies for the various components, and with a direct converter efficiency of 90%, a thermal converter efficiency of 45%, and a perfectly efficient injector the critical Q value is found to be 1.22. Using this Q value, the physical dimensions of the system along with the performance parameters can be obtained by solving an appropriate set of conservation equations that utilize the gasdynamic confinement time τ, given by

$$\tau = RL/v_{th}$$

where R is the mirror ratio seen by the plasma, and v_{th} is the mean velocity of the escaping particles. The results obtained in Refs. 1 and 2 do not account for the presence of the electrostatic potential, which arises as a result of the fast escape of the electrons because of their small mass. As the electrons escape, they leave behind an excess of positive charge, which manifests itself as a positive electric potential that slows down the electron escape while speeding up the ions until their respective axial diffusions are equalized. The indirect effect on the ions is that their confinement time is reduced, and to compensate for that, the length must increase to allow for recovery of an equal amount of fusion power. But as they emerge from the thruster mirror, the ions receive an added energy equal to the potential, and that manifests itself in increased specific impulse and thrust. In the remainder of this paper, we present the mathematical formulation of the electrostatic potential, and examine its effect on the dynamics of the plasma and the concomitant impact on the propulsive performance of GDM.

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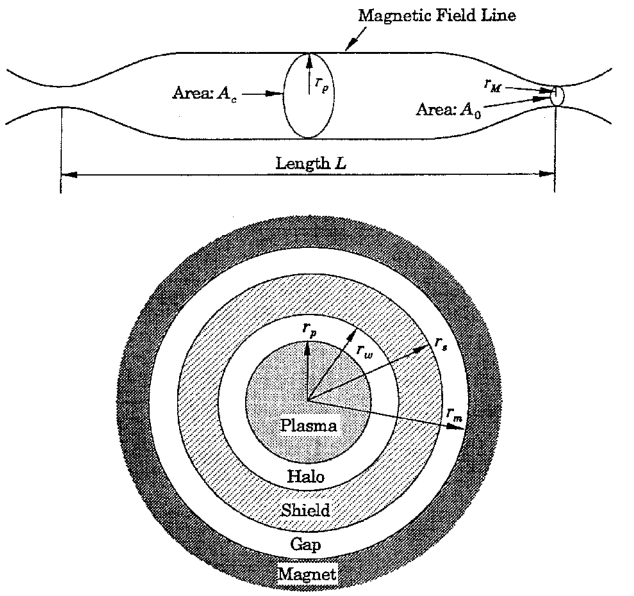

Fig. 1 Schematic and cross-sectional view of the gasdynamic fusion propulsion system.

Mathematical Derivation and Analysis

We begin with the monoenergetic diffusion equations for the electrons and ions in the device[3]:

$$(1/R)\gamma_e = -D_e \nabla n_e - \mu_e \underset{>}{E} n_e \tag{1}$$

$$(1/R)\gamma_i = -D_i \nabla n_i + \mu_i z \underset{>}{E} n_i \tag{2}$$

In Eqs. (1) and (2), R reflects the fact that γ is measured at the throat of the mirror, where the area A_M is reduced by a factor of R relative to the central area (or region), where the right-hand sides are calculated, i.e., $A_0 = A_c/R$. It is assumed that the ion and electron densities vary as

$$n = n_0 \exp[-(2k/L)z] \tag{3}$$

where L is the axial length of the system and k is an integer. It is clear from this equation that the density gradient is given by

$$\nabla n = -(2k/L)n \tag{4}$$

which can be employed in writing the total monoenergetic flux through the mirror as

$$A_M \gamma = -AD\nabla n = (2k/L)ADn \tag{5}$$

The total loss through both mirrors (assuming symmetry) can be expressed in terms of a loss time constant τ as

$$2A_M \gamma = V(n/\tau) = AL(n/\tau) \tag{6}$$

From Eqs. (5) and (6), we readily note that

$$D = L^2/4k\tau \tag{7}$$

For the ions the loss time constant noted earlier is used, namely,

$$\tau_i = RL/v_i \tag{8}$$

where v_i is the velocity of the (monoenergetic) ions. Thus, the ion diffusion coefficient D_i can be expressed by

$$D_i = \frac{L^2}{4k\tau_i} = \frac{L^2}{4k}\left(\frac{v_i}{RL}\right) = \frac{Lv_i}{4Rk} \tag{9}$$

For the electrons the diffusion coefficient is used, and it is given by[3]

$$D_e = v_e^2/3\upsilon_{ei} \tag{10}$$

In addition, the following definitions of the electron and ion mobilities are employed:

$$\mu_e = e/m_e\upsilon_{ei} \tag{11}$$

$$\mu_i = Ze/m_i\upsilon_{ei} \tag{12}$$

where υ_{ei} is the electron–ion collision frequency, represented by[4]

$$\upsilon_{ei} = \frac{n_e \,\ell n\, \lambda}{C_0 E_e^{3/2}} \tag{13}$$

with the constant $C_0 = 8.176 \times 10^9$ s cm^{-3} keV$^{-3/2}$.

It is further assumed that the electrostatic potential varies in space in the same manner as the electron and ion densities, so that the electric field becomes

$$\underset{>}{E} = -\nabla\phi = (2k/L)\phi \tag{14}$$

With that, the monoenergetic fluxes given by Eqs. (1) and (2) now become

$$\gamma_e = \frac{2Rk}{m_e L\upsilon_{ei}}\left(\frac{1}{3}m_e v_e^2 - e\phi\right) n_e \tag{15}$$

$$\gamma_i = \frac{1}{2}\left(v_i + \frac{4Rk}{m_e L\upsilon_{ei}}\frac{m_e}{m_i} z_i^2 e\phi\right) n_i \tag{16}$$

Note that for small v_e, the electron flux γ_e becomes negative. Because this is physically unrealistic (there is no source outside the reactor to produce this return flow), we set $\gamma_e = 0$ for $v_e < v_m$, where

$$v_m = \sqrt{3e\phi/m_e} \tag{17}$$

Because γ_i is always positive, the lower limit on Eq. (16) is zero.

Because the plasma in GDM is highly collisional, it is reasonable to assume that the species have Maxwell–Boltzmann velocity distributions, and, with that, the total electron and ion fluxes can be found by integrating Eqs. (15) and (16) over all velocities. Thus, we have

$$\Gamma_e = \frac{2Rk}{m_e L\upsilon_{ei}}\frac{4}{\sqrt{\pi}}\left(\frac{m_e}{2T_e}\right)^{3/2} \times N_e \int_{v_m}^{\infty}\left(\frac{1}{3}m_e v^4 - e\phi v^2\right)\exp\left(-\frac{m_e}{2T_e}v^2\right)\mathrm{d}v \tag{18}$$

$$\Gamma_i = \frac{1}{2}\frac{4}{\sqrt{\pi}}\left(\frac{m_i}{2T_i}\right)^{3/2} \times N_i \int_0^{\infty}\left(v^3 + \frac{4Rk}{m_e L\upsilon_{ei}}\frac{m_e}{m_i} z_i^2 e\phi v^2\right)\exp\left(-\frac{m_i v^2}{2T_i}\right)\mathrm{d}v \tag{19}$$

where T_e and T_i are the electron and ion temperatures, respectively. The integrals in the preceding expressions can be readily carried out, and after some algebra we obtain

$$\Gamma_e = \frac{2RkN_eT_e}{m_eL\upsilon_{\rm ei}}\left\{\left(1 - \frac{e\phi}{T_e}\right)\left[1 - erf\left(\sqrt{\frac{3e\phi}{2T_e}}\right)\right] + \frac{2}{\sqrt{\pi}}\sqrt{\frac{3e\phi}{2T_e}}\exp\left(-\frac{3e\phi}{2T_e}\right)\right\} \tag{20}$$

$$\Gamma_i = \frac{2RkN_iT_i}{m_eL\upsilon_{\rm ei}}\left[\frac{m_eL\upsilon_{\rm ei}}{4RkT_i}\left(\frac{8T_i}{\pi m_i}\right)^{1/2} + \frac{m_e}{m_i}z_i^2\frac{e\phi}{T_i}\right] \tag{21}$$

where $erf(y) = 2/\sqrt{\pi}\int_0^y e^{-t^2}\,dt$ is the familiar error function, and z_i is the ion charge number.

The condition of charge neutrality requires that the charged flux losses be equal and the net charge be zero, or

$$\Gamma = \Gamma_e = z_i\Gamma_i \tag{22}$$

$$N = N_e = z_iN_i \tag{23}$$

This means that when we multiply Eq. (21) by z_i and equate the result to Eq. (20), and then make use of Eq. (23) to eliminate the densities, we obtain a balance equation that the ambipolar potential ϕ must satisfy; it is

$$\left(1 - \frac{e\phi}{T_e}\right)\left[1 - erf\left(\sqrt{\frac{3e\phi}{2T_e}}\right)\right] + \frac{2}{\sqrt{\pi}}\sqrt{\frac{3e\phi}{2T_e}}\exp\left(-\frac{3e\phi}{2T_e}\right) = \frac{m_eL\upsilon_{\rm ei}}{4RkT_e}\left(\frac{8T_i}{\pi m_i}\right)^{1/2} + \frac{m_e}{m_i}z_i^2\frac{e\phi}{T_e} \tag{24}$$

The appearance of this balance equation can be simplified with the use of the following two parameters:

$$\delta = \frac{L}{4Rk}m_e\upsilon_{\rm ei}\left(\frac{8T_i}{\pi m_i}\right)^{1/2} \tag{25}$$

$$x = \sqrt{3e\phi/2T_e} \tag{26}$$

The parameter δ has units of energy, while x is clearly dimensionless. Using these parameters, Eq. (24) can be rewritten as

$$\left(1 - \frac{2}{3}x^2\right)[1 - erf(x)] + \frac{2}{\sqrt{\pi}}x\exp(-x) = \frac{\delta}{T_e} + \frac{2}{3}\frac{m_e}{m_i}z_i^2x^2 \tag{27}$$

It can be shown that the left-hand side of Eq. (27) decreases monotonically from one when $x = 0$ to zero when $x = \infty$. Hence, there is always a solution x, provided that $\delta \le T_e$.

Returning to Eq. (7), the time constant can be expressed by

$$\tau = L^2/4kD \tag{28}$$

In Eq. (28), the diffusion coefficient D is found from the condition

$$(1/R)\Gamma = -D\nabla N = (2k/L)DN \tag{29}$$

and using Eqs. (21–23), we obtain

$$D = \frac{T_i}{m_e\upsilon_{\rm ei}}\left[\frac{m_eL\upsilon_{\rm ei}}{4RkT_i}\left(\frac{8T_i}{\pi m_i}\right)^{1/2} + \frac{m_e}{m_i}z_i^2e\phi\right] = \frac{1}{m_e\upsilon_{\rm ei}}\left(\delta + \frac{m_e}{m_i}z_i^2e\phi\right) \tag{30}$$

If the velocity-averaged ion-confinement time is now defined as

$$\tau_i \equiv RL\sqrt{\pi m_i/8T_i} \tag{31}$$

then, from Eq. (25), it is found

$$\delta = \frac{L^2m_e\upsilon_{\rm ei}}{4k\tau_i} \tag{32}$$

and that makes Eq. (30) assume the following form:

$$D = \frac{L^2}{4k\tau_i}\left(1 + \frac{m_e}{m_i}z_i^2\frac{e\phi}{\delta}\right) \tag{33}$$

This in turn makes the time constant given by Eq. (28) become

$$\tau = \frac{\tau_i}{1 + (m_e/m_i)z_i^2(e\phi/\delta)} \tag{34}$$

We note at this juncture that when ϕ is neglected, the characteristic confinement time reduces to that given by Eq. (31), which was utilized in previous studies of GDM.[1,2]

In addition to the confinement properties obtained in the preceding text, it is necessary in assessing the propulsive capability of GDM to calculate the average escape energies of the charged particles of the system. These can be obtained by multiplying Eqs. (1) and (2) by the appropriate kinetic energies and then integrating the results over the velocity distributions. Thus, we can write

$$\frac{1}{R}\Gamma_eE_{\rm Le} = \frac{2k}{L}\frac{4}{\sqrt{\pi}}\left(\frac{m_e}{2T_e}\right)^{3/2} \times N_e\int_{v_m}^{\infty}\left(\frac{m_e}{6\upsilon_{\rm ei}}v^6 - \frac{1}{2}m_e\mu_e\phi v^4\right)\exp\left(-\frac{m_ev^2}{2T_e}\right)dv \tag{35}$$

$$\frac{1}{R}\Gamma_iE_{\rm Li} = \frac{2k}{L}\frac{4}{\sqrt{\pi}}\left(\frac{m_i}{2T_i}\right)^{3/2} \times N_i\int_0^{\infty}\left(\frac{Lm_i}{8Rk}v^5 + \frac{1}{2}m_i\mu_iz_i\phi v^4\right)\exp\left(-\frac{m_iv^2}{2T_i}\right)dv \tag{36}$$

These integrals can be evaluated in the same manner as the previous ones, and after some algebraic manipulations we obtain

$$\Gamma_eE_{\rm Le} = \frac{2RkN_eT_e}{m_eL\upsilon_{\rm ei}}\left\{\frac{1}{2}\left(5 - \frac{3}{2}\frac{e\phi}{T_e}\right)\left[1 - erf\left(\sqrt{\frac{3e\phi}{2T_e}}\right)\right] + \frac{1}{\sqrt{\pi}}\left(5 + \frac{2e\phi}{T_e}\right)\sqrt{\frac{3e\phi}{2T_e}}\exp\left(-\frac{3e\phi}{2T_e}\right)\right\}T_e \tag{37}$$

$$\Gamma_iE_{\rm Li} = \frac{N_iT_i}{R}\left(\frac{8T}{\pi m_i}\right)^{1/2} + \frac{3kN_iz_i^2T_i}{Lm_i\upsilon_{\rm ei}}e\phi \tag{38}$$

If we now use Eqs. (20) and (21), and utilize the definitions given in Eqs. (25) and (26), the expressions for the electron and ion escape energies are as follows:

$$E_{\rm Le} = \left\{\frac{(5 - 2x^2)[1 - erf(x)] + \frac{2}{\sqrt{\pi}}(5 + \frac{4}{3}x^2)x\exp(-x^2)}{2(1 - \frac{2}{3}x^2)[1 - erf(x)] + (4/\sqrt{\pi})x\exp(-x^2)}\right\}T_e \tag{39}$$

$$E_{\rm Li} = \left[\frac{2 + (m_e/m_i)z_i^2(T_e/\delta)x^2}{1 + \frac{2}{3}(m_e/m_i)z_i^2(T_e/\delta)x^2}\right]T_i \tag{40}$$

It can be observed once again from Eq. (40) that when the potential is ignored, the ion escape energy of $2T_i$ used in previous calculations is recovered. Also note that the calculated quantities E_{Le} and E_{Li} are the average energies of escaping electrons and ions as they leave the plasma chamber. The potential must then be added to the ion energy and subtracted from the electron energy to obtain the energies of these particles outside the chamber. Thus, the average energy of an escaping electron outside the plasma is $(E_{Le} - e\phi)$, whereas that of an escaping ion is $(E_{Li} + e\phi)$.

Application to GDM Thruster

The preceding results are now applied to the gasdynamic mirror propulsion system, whose properties in the absence of the potential were obtained earlier.[2] Those parameters will be repeated here for the sake of comparison, and to put them in the proper perspective it is desirable to briefly review how they were obtained. On the assumption that half of the charged particle power appears as thrust power while the other half goes to the direct converter with an efficiency of 90%, a power flow diagram is used to calculate the Q value of 1.22. This value was obtained assuming that the radiated power (of bremsstrahlung and synchrotron radiation) along with the neutron power generated by these fusion reactions are processed by a thermal converter at an efficiency of 45%. A hydrodynamically stable plasma was assumed to be confined by a central magnetic field at an efficiency of 95%, implying that the ratio of the plasma pressure to the magnetic field pressure as given by the parameter β is $\beta = 0.95$. The performance of the system was assumed to be determined by the ions so that, with the exception of the radiative losses, the electron dynamics were ignored in the balance equations that characterize the system. Hence, the equations utilized were the ion mass and energy conservation equations, with the latter modified to account for the radiative losses. With the confinement time represented by Eq. (8), this system of equations can be solved for the length of the device along with other parameters that reflect not only the plasma behavior, but also the propulsive capability of GDM. Although Refs. 1 and 2 should be consulted for these details, $e\phi = 0$ in Table 1 gives the relevant parameters when the electrostatic potential is ignored. For the example shown, a gain factor of 1.222 was obtained assuming the efficiencies noted earlier, and assuming the charged particle power to be evenly divided between the thrust power and the power to the direct converter.

Because the electrostatic potential is inherent to plasma confinement in mirror machines and closely coupled to the electron dynamics as demonstrated in the previous analysis, it is clear that a more realistic assessment of GDM must include both of these effects. When these effects are taken into account and the appropriate equations are solved, the performance parameters given in column 2 of Table 1 are obtained. At the same density and ion temperature as those given in column 1 of Table 1, it is seen that the confinement time for both species is indeed longer than in the approximate model. As can be seen from Eqs. (31) and (34), this means an increase in the length and correspondingly the volume of the plasma that, in turn, means larger fusion and thrust powers. Another important consequence is the large electron escape energy (~187 keV), which reflects, on the one hand, a major contribution to the thrust power, and on the other, a significant loss of energy from the reactor that manifests itself in the larger engine volume needed to compensate for that loss. As the ions emerge from the nozzle they are accelerated by the potential and, as a result, give rise to a larger specific impulse of about 1.60×10^5 s compared to 1.27×10^5 s when these effects are ignored. The thrust is correspondingly increased almost threefold, and while the total vehicle mass more than doubled, the round-trip time to Mars in fact decreased by about 12 days. This enhancement in the propulsive performance of GDM is further represented by a specific power of 15.7 kW/kg as compared to 13.4 kW/kg in the approximate case. This more accurate analysis of GDM also reveals that the electron temperature in the reactor is significantly lower than that of the ion, namely, 2.61 keV compared to 10 keV, and because the density of both species is the same, the confining magnetic field is smaller in the case of $e\phi \neq 0$. Although this results in smaller magnet masses, the engine mass and total mass are bigger in this case because of the increase in length that, in turn, results in larger fusion and thrust powers.

The larger vehicle mass predicted by the realistic physics model may cast a shadow on the potential of GDM as a propulsion device despite a better performance. This also is not an accurate representation because no attempt was made to optimize the system. Any attempt in this regard would be quite extensive because of the many parameters that play a role in determining the outcome. A partial attempt in this direction is

Table 1 GDM system for deuterium–tritium fuel cycle

Parameter	$e\phi = 0$	$e\phi \neq 0$
Plasma density	10^{16} cm^{-3}	10^{16} cm^{-3}
Plasma temperature	10 keV	10 keV
Plasma mirror ratio	100	100
β value	0.95	0.95
Central magnetic field	9.207 T	6.506 T
Plasma radius	5 cm	5 cm
Gain factor Q	1.222	1.222
Plasma length	43.708 m	118.337 m
Confinement time	4.068×10^{-3} s	1.194×10^{-2} s
Electrostatic potential	0	11.446 keV
Thrust	2.512×10^3 N	7.428×10^3 N
Thrust power	1.351×10^3 MW	3.745×10^3 MW
Fusion power	2.730×10^3 MW	7.387×10^3 MW
Bremsstrahlung power	58.17 MW	27.40 MW
Synchrotron power	18.94 MW	3.314 MW
Neuton power	2.183×10^3 MW	5.910×10^3 MW
Reactor mass	55.50 mg	131.2 mg
Injector mass	45.40 mg	107.3 mg
Engine mass (Reactor and injector)	100.90 mg	238.5 mg
Thermal converter mass	45.90 mg	104.9 mg
Direct converter mass	27.50 mg	66.50 mg
Radiator mass	248.60 mg	653.4 mg
Total vehicle mass	422.90 mg	1063.3 mg
Specific impulse	1.268×10^5 s	1.604×10^5 s
Specific power	13.399	15.7
Trip time	169 days	157 days

Table 2 Performance characteristics of a GDM rocket[a]

T, keV	Radius, cm	Central field, T	Length, m	Isp, s	Thrust, N	Total mass, mg	Mars round-trip, days
10	3.164	6.506	118.3	1.604×10^5	2.975×10^3	650.5	193.5
12	2.908	7.127	90.5	1.681×10^5	2.936×10^3	564.3	181.4
14	2.688	7.698	76.4	1.762×10^5	2.871×10^3	522.1	176.4
16	2.497	8.230	68.9	1.844×10^5	2.795×10^3	501.2	175.2
18	2.332	8.729	65.0	1.926×10^5	2.714×10^3	492.3	176.0
20	2.188	9.201	63.3	2.006×10^5	2.634×10^3	490.9	178.3

[a] $n = 10^{16}$ cm^{-3}, $R = 100$, $P_{th} = 1.5 \times 10^3$ MW.

displayed in Table 2, where we have focused on a system that delivers a thrust power of 1.5×10^3 MW at a density of 10^{16} cm^{-3}, and a mirror ratio of 100 while varying the ion temperature from 10 to 20 keV, and the plasma radius from about 3.2 cm to approximately 2.2 cm. We observe that the shortest trip time is achieved at a temperature of 16 keV, whereas the shortest system with the smallest mass is achieved at an ion temperature of 20 keV. The larger magnets required in this case is more than offset by the shorter length; and the slight reduction in the thrust is much more than offset by a specific impulse that far exceeds 2.0×10^5 s. When compared to the approximate model described in Table 1, it is clear that a GDM operating at 20-keV temperature generates effectively the same physical and operating parameters, but on the basis of a more rigorous physical foundation.

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